RESEARCH ARTICLE

On Optimization of Manufacturing of Field-effect Transistors in the Framework of a Switch to Increase their Integration rate-On Influence Mismatch-induced Stress

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ABSTRACT: In this article, we describe a method to increase the density of field-effect transistors in the framework of an absorptive single pole single throw switch. In the framework of the approach we consider to manufacturing of inverters in heterostructure with specific configuration. Several required areas of the heterostructure should be doped by diffusion or ion implantation. After that, the dopant and radiation defects should by annealed in the framework of optimized scheme. We obtain dependences of optimal value of annealing time on several parameters. We also considered a method to reduce the amount of stress mismatch in the considered heterostructure. We consider an analytical approach to analyze mass and heat transport in heterostructures during manufacturing of integrated circuits with account mismatch-induced stress.

KEYWORDS: absorptive single pole single throw switch; optimization of manufacturing; analytical approach for prognosis.

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INTRODUCTION

In the present time several actual problems of the solid state electronics such as increasing of performance, reliability and density of elements of integrated circuits: diodes, field-effect and bipolar transistors are intensively solving [1-6]. To increase the performance of these devices it is attracted an interest to determine the materials with higher values of charge carrier mobility [7-10]. One way to decrease the dimensions of the elements of integrated circuits is manufacturing them in thin film heterostructures [3-5,11]. In this case, it is possible to use inhomogeneity of heterostructure and necessary optimization of doping of electronic materials [12,13] and development of epitaxial technology to improve these materials (including analysis of mismatch induced stress) [14-16]. An alternative approach to increase the dimensions of integrated circuits are using of laser and microwave types of annealing [17-19].

In the framework of the paper we introduce an approach to manufacture field-effect transistors. The approach gives a possibility to decrease their dimensions with increasing their density framework a absorptive single pole single throw switch. We also consider the possibility to decrease mismatch-induced stress to decrease the quantity of defects, generated due to the stress. In this paper, we consider a heterostructure, which consist of a substrate and an epitaxial layer (see Fig. 1). We also consider a buffer layer between the substrate and the epitaxial layer. The epitaxial layer includes into itself several sections, which were manufactured by using other materials. These sections have been doped by diffusion or ion implantation to manufacture the required types of conductivity (*p* or *n*). These areas became sources, drains and gates (see Fig. 1). After doping it is required to annealing of dopant and/or radiation defects.

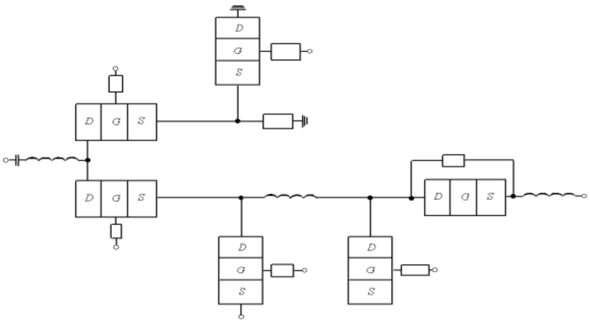

Fig. 1*a*: Structure of the considered switch [15]

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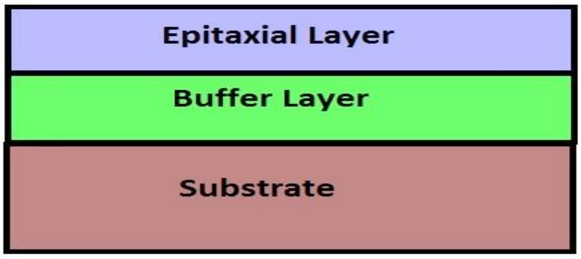

Fig 1*b*: Heterostructure with a substrate, epitaxial layers and buffer layer (view from side)

The main aim of the present paper is to analysis of redistribution of dopant and radiation defects to determine conditions, which correspond to decreasing of elements of the considered filter and at the same time to increase their density. At the same time we consider a possibility to decrease mismatch-induced stress.

Method of Solutions

To solve our aim we calculate spatio-temporal distribution of concentration of dopant in the considered heterostructure and analyzed it. The above distribution was obtained as solution of the following the second Fick's law [1,20-24].

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+ \tag{1}$$

$$\Omega\left\{\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]\right\}$$

with boundary and initial conditions

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ C(x,y,z,0)=f_C(x,y,z).$$

Here $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant; Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient; $\int_0^{L_z}C(x,y,z,t)\,dz$ is the surficial concentration of dopant on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure); $\mu_1(x,y,z,t)$ is the chemical potential due to the presence of mismatch-induced stress; D and D_S are the coefficients of volumetric and surficial diffusions. The values of dopant diffusion coefficients depends on the properties of materials of heterostructure, the speed of heating and cooling of materials during annealing and spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations [22-24]

$$D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$

$$D_S=D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \tag{2}$$

Here D_L (x,y,z,T) and D_{LS} (x,y,z,T) are the spatial (due to accounting all layers of heterostruicture) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficients; T is the temperature of annealing; P (x,y,z,T) is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer in the following interval $\gamma \in[1,3]$ [22]; V (x,y,z,t) is the spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of vacancies. Concentrational dependence of dopant diffusion coefficient has been described in details in [22]. Spatio-temporal distributions of concentration of point radiation defects have been determined by solving the following system of equations [20,23,24]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}I(x,y,W,t)\,dW\right]$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)-k_{I,V}(x,y,z,T)\,I(x,y,z,t)\,V(x,y,z,t)$$

$$+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}I(x,y,W,t)\,dW\right] \qquad (3)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}V(x,y,W,t)\,dW\right]$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)-k_{I,V}(x,y,z,T)\,I(x,y,z,t)\,V(x,y,z,t)$$

$$+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}V(x,y,W,t)\,dW\right]$$

with boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$I(x,y,z,0)=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z). \qquad (4)$$

Here I (x,y,z,t) is the spatio-temporal distribution of concentration of radiation interstitials; I^* is the equilibrium distribution of interstitials; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y, z,T)$, $D_{VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials, respectively (see, for example, [24] and appropriate references in this book); $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point radiation defects and generation of their complexes.

Spatio-temporal distributions of divacancies Φ_V (x,y,z,t) and diinterstitials Φ_I (x,y,z,t) could be determined by solving the following system of equations [20,23,24]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right] \tag{5}$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+k_I(x,y,z,T)I(x,y,z,t)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z). \tag{6}$$

Here $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential μ_1 in Eq.(1) could be determine by the following relation [20]

$$\mu_1=E(z)\Omega\sigma_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2, \tag{7}$$

where $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The Eq. (3) could be transform to the following form

$$\mu(x,y,z,t)=E(z)\frac{\Omega}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)[T(x,y,z,t)-T_0]\,\delta_{ij}\right\}, \tag{8}$$

where σ is Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincide (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [25]

$$\begin{cases} \rho(z)\dfrac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{xx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{xy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{xz}(x,y,z,t)}{\partial z} \\ \rho(z)\dfrac{\partial^2 u_y(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{yx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{yy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{yz}(x,y,z,t)}{\partial z} \\ \rho(z)\dfrac{\partial^2 u_z(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{zx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{zy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{zz}(x,y,z,t)}{\partial z} \end{cases} \tag{9}$$

where

$$\sigma_{ij} = \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j} + \frac{\partial u_j(x,y,z,t)}{\partial x_i} - \frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right] + K(z)\delta_{ij}\frac{\partial u_k(x,y,z,t)}{\partial x_k}$$
$$-\beta(z)K(z)[T(x,y,z,t)-T_r],$$

$\rho(z)$ is the density of materials of heterostructure, δ_{ij} Is the Kronecker symbol. With account the relation for σ_{ij} last system of equation could be written as

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = \left\{K(z) + \frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2} + \left\{K(z) - \frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial x \partial y}$$
$$+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right] + \left[K(z) + \frac{E(z)}{3[1+\sigma(z)]}\right]\frac{\partial^2 u_z(x,y,z,t)}{\partial x \partial z}$$
$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2} = \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u_x(x,y,z,t)}{\partial x \partial y}\right] - K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}$$
$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z} + \frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\} + \frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]} + K(z)\right\} \tag{10}$$
$$+\left\{K(z) - \frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y \partial z} + K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x \partial y}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2} = \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u_z(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u_x(x,y,z,t)}{\partial x \partial z} + \frac{\partial^2 u_y(x,y,z,t)}{\partial y \partial z}\right]$$
$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z} - \frac{\partial u_x(x,y,z,t)}{\partial x} - \frac{\partial u_y(x,y,z,t)}{\partial y} - \frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}$$
$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z} + \frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x} + \frac{\partial u_y(x,y,z,t)}{\partial y} + \frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}.$$

Conditions for the system of Eq. (8) could be written in the form

$$\frac{\partial \vec{u}(0,y,z,t)}{\partial x}=0\,;\ \frac{\partial \vec{u}(L_x,y,z,t)}{\partial x}=0\,;\ \frac{\partial \vec{u}(x,0,z,t)}{\partial y}=0\,;\ \frac{\partial \vec{u}(x,L_y,z,t)}{\partial y}=0\,;\ \frac{\partial \vec{u}(x,y,0,t)}{\partial z}=0\,;$$

$$\frac{\partial \vec{u}(x,y,L_z,t)}{\partial z}=0\,;\ \vec{u}(x,y,z,0)=\vec{u}_0\,;\ \vec{u}(x,y,z,\infty)=\vec{u}_0\,.$$

We determine spatio-temporal distributions of concentrations of dopant and radiation defects by solving the Eqs.(1), (3) and (5) using standard method of averaging of function corrections [26]. Previously, we transform the Eqs.(1), (3) and (5) to the following form with account initial distributions of the considered concentrations:

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]$$
$$+f_C(x,y,z)\delta(t) \qquad (1a)$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]$$
$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$
$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_I(x,y,z)\delta(t) \qquad (3a)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]$$
$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$
$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_V(x,y,z)\delta(t)$$

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t)$$
$$+k_{I,I}(x,y,z,T)\,I^2(x,y,z,t) \qquad (5a)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_V(x,y,z,T)V(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t)$$
$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

Farther, we replaced the concentration of dopants and radiation defects in right sides of Equations (1a), (3a) and (5a), on their not yet known average values $\alpha_{1\rho}$. In this situation, we obtain equations for the first-order approximations of the required concentrations in the following form

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]$$
$$+f_C(x,y,z)\delta(t) \qquad (1b)$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+f_I(x,y,z)\delta(t)$$
$$-\alpha_{1I}^2k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \qquad (3b)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_V(x,y,z)\delta(t)$$
$$-\alpha_{1V}^2k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]$$
$$+f_{\Phi_I}(x,y,z)\delta(t)+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \qquad (5b)$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]$$
$$+f_{\Phi_V}(x,y,z)\delta(t)+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

Integration on the left and right sides of the Equation (1b), (3b) and (5b) on time gives us possibility to obtain relations for above approximation in the final form

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int\limits_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left[1+\frac{\xi_S\alpha_{1C}^{\gamma}}{P^{\gamma}(x,y,z,T)}\right]\cdot$$
$$\cdot\nabla_S\mu_1(x,y,z,\tau)\,d\tau\,\}+\alpha_{1C}\Omega\frac{\partial}{\partial y}\int\limits_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\cdot$$
$$\cdot\nabla_S\mu_1(x,y,z,\tau)\frac{z}{kT}\left[1+\frac{\xi_S\alpha_{1C}^{\gamma}}{P^{\gamma}(x,y,z,T)}\right]d\tau+f_C(x,y,z) \qquad (1c)$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+f_I(x,y,z)$$

$$-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau \qquad (3c)$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+f_V(x,y,z)$$

$$-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\Omega\alpha_{1\Phi_I}z\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau$$

$$+f_{\Phi_I}(x,y,z)+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau+\int_0^t k_{I,I}(x,y,z,T)\,I^2(x,y,z,\tau)d\tau \qquad (5c)$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\Omega\alpha_{1\Phi_V}z\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau$$

$$+f_{\Phi_V}(x,y,z)+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)d\tau.$$

We determine average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [26]

$$\alpha_{1\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t. \qquad (11)$$

Substitution of the relations (1*c*), (3*c*) and (5*c*) into relation (9) gives us possibility to obtain required average values in the following form

$$\alpha_{1C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)d\,z\,d\,y\,d\,x,\ \alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+\Theta^2L_xL_yL_za_1}{a_4}\right)}-\frac{a_3+A}{4a_4},$$

$$\alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z\right],$$

where $$S_{\rho\rho ij}=\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t,$$

$$a_4=S_{II00}\times\left(S_{IV00}^2-S_{II00}S_{VV00}\right),$$

$$a_3=S_{IV00}S_{II00}+S_{IV00}^2-S_{II00}S_{VV00},\ a_2=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)d\,z\,d\,y\,d\,x\times$$

$$\times S_{IV00}S_{IV00}^2+S_{IV00}\Theta L_x^2L_y^2L_z^2+2S_{VV00}S_{II00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x-\Theta L_x^2L_y^2L_z^2S_{VV00}$$

$$-S_{IV00}^{2}\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z} f_I\left(x,y,z\right)d\,z\,d\,y\,d\,x,$$

$$a_1=S_{IV00}\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z} f_I\left(x,y,z\right)d\,z\,d\,y\,d\,x,\ a_0=S_{VV00}$$

$$\times\left[\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z} f_I\left(x,y,z\right)d\,z\,d\,y\,d\,x\right]^2$$

$$A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\ B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}$$

$$-\sqrt[3]{\sqrt{q^2+p^3}+q},\ q=\frac{\Theta^3 a_2}{24a_4^2}\left(4a_0-\Theta L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)$$

$$-\frac{\Theta^3a_2^3}{54a_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2},\ p=\Theta^2\frac{4a_0a_4-\Theta L_xL_yL_za_1a_3}{12a_4^2}-\frac{\Theta a_2}{18a_4},$$

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z} f_{\Phi_I}\left(x,y,z\right)d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z} f_{\Phi_V}\left(x,y,z\right)d\,z\,d\,y\,d\,x,$$

where $R_{\rho i}=\int_{0}^{\Theta}\left(\Theta-t\right)\int_{0}^{L_x}\int_{0}^{L_y}\int_{0}^{L_z}k_I\left(x,y,z,T\right)I_1^i\left(x,y,z,t\right)d\,z\,d\,y\,d\,x\,d\,t$.

We also determine approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of method of averaging of function corrections [26]. Framework this procedure to determine approximations of the *n*-th order of concentrations of dopant and radiation defects we replace the required concentrations in the Eqs. (1*c*), (3*c*), (5*c*) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations

$$\frac{\partial C_2\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,t\right)\right]^{\gamma}}{P^{\gamma}\left(x,y,z,T\right)}\right\}\left[1+\varsigma_1\frac{V\left(x,y,z,t\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,t\right)}{\left(V^*\right)^2}\right]\cdot\right.$$

$$\left.\cdot D_L\left(x,y,z,T\right)\frac{\partial C_1\left(x,y,z,t\right)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V\left(x,y,z,t\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,t\right)}{\left(V^*\right)^2}\right]\frac{\partial C_1\left(x,y,z,t\right)}{\partial y}\cdot\right.$$

$$\left.\cdot D_L\left(x,y,z,T\right)\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,t\right)\right]^{\gamma}}{P^{\gamma}\left(x,y,z,T\right)}\right\}\right)+\frac{\partial}{\partial z}\left(\left[1+\varsigma_1\frac{V\left(x,y,z,t\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,t\right)}{\left(V^*\right)^2}\right]\cdot\right.$$

$$\cdot\, D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\biggl\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^{\gamma}}{P^{\gamma}(x,y,z,T)}\biggr\}\biggr)+f_C(x,y,z)\delta(t)$$

$$+\Omega\frac{\partial}{\partial x}\biggl\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,dW\biggr\}$$

$$+\Omega\frac{\partial}{\partial y}\biggl\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,dW\biggr\} \quad (1d)$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\biggl[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\biggr]+\frac{\partial}{\partial y}\biggl[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\biggr]$$

$$+\frac{\partial}{\partial z}\biggl[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\biggr]-k_{I,V}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]$$

$$-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2+\Omega\frac{\partial}{\partial x}\biggl\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\biggr\}$$

$$+\Omega\frac{\partial}{\partial y}\biggl\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\biggr\} \qquad (3d)$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\biggl[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\biggr]+\frac{\partial}{\partial y}\biggl[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\biggr]$$

$$+\frac{\partial}{\partial z}\biggl[D_I(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\biggr]-k_{I,V}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]$$

$$-k_{V,V}(x,y,z,T)[\alpha_{1V}+V_1(x,y,z,t)]^2+\Omega\frac{\partial}{\partial x}\biggl\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\biggr\}$$

$$+\Omega\frac{\partial}{\partial y}\biggl\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\biggr\}$$

$$\frac{\partial \Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\biggl[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial x}\biggr]+\frac{\partial}{\partial y}\biggl[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial y}\biggr]$$

$$+\Omega\frac{\partial}{\partial x}\biggl\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)]\,dW\biggr\}+k_{I,I}(x,y,z,T)I^2(x,y,z,t)$$

$$+\Omega\frac{\partial}{\partial y}\biggl\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)]\,dW\biggr\}+k_I(x,y,z,T)I(x,y,z,t)$$

$$+\frac{\partial}{\partial z}\biggl[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial z}\biggr]+f_{\Phi_I}(x,y,z)\delta(t) \qquad (5d)$$

$$\frac{\partial \Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\biggl[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial x}\biggr]+\frac{\partial}{\partial y}\biggl[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial y}\biggr]$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+k_V\left(x,y,z,T\right)V\left(x,y,z,t\right)$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial z}\right]+f_{\Phi_V}\left(x,y,z\right)\delta\left(t\right).$$

Integration of the left and the right sides of Equations (1*d*), (3*d*) and (5*d*) give us possibility to obtain relations for the required concentrations in the final form

$$C_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\cdot$$

$$\cdot D_L\left(x,y,z,T\right)\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_L\left(x,y,z,T\right)\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\cdot$$

$$\cdot\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial y}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,t\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}+\frac{\partial}{\partial z}\int\limits_0^t\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\cdot$$

$$\cdot D_L\left(x,y,z,T\right)\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial z}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}d\tau+\Omega\frac{\partial}{\partial y}\int\limits_0^t\nabla_S\mu\left(x,y,z,\tau\right)\frac{D_S}{kT}\cdot$$

$$\cdot\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau$$

$$+f_C\left(x,y,z\right)\tag{1e}$$

$$I_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t D_I\left(x,y,z,T\right)\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_I\left(x,y,z,T\right)\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial y}d\tau$$

$$+\frac{\partial}{\partial z}\int\limits_0^t D_I\left(x,y,z,T\right)\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial z}d\tau-\int\limits_0^t k_{I,I}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]^2d\tau$$

$$-\int\limits_0^t k_{I,V}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]d\tau+\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\cdot$$

$$\cdot\int\limits_0^{L_z}\left[\alpha_{2I}+I_1\left(x,y,W,\tau\right)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1\left(x,y,W,\tau\right)\right]dW\,d\tau\tag{3e}$$

$$V_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t D_V\left(x,y,z,T\right)\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_V\left(x,y,z,T\right)\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial y}d\tau$$

$$+\frac{\partial}{\partial z}\int\limits_0^t D_V\left(x,y,z,T\right)\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial z}d\tau-\int\limits_0^t k_{V,V}\left(x,y,z,T\right)\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]^2d\tau$$

$$-\int\limits_0^t k_{I,V}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]d\tau+\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_{VS}}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\cdot$$

$$\cdot\int\limits_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]d\,W\,d\,\tau+\Omega\frac{\partial}{\partial\,y}\int\limits_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,\tau)\int\limits_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]d\,W\,d\,\tau\,.$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial\,x}\int\limits_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\,\Phi_{1I}(x,y,z,\tau)}{\partial\,x}d\,\tau+\frac{\partial}{\partial\,y}\int\limits_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\,\Phi_{1I}(x,y,z,\tau)}{\partial\,y}d\,\tau$$

$$+\frac{\partial}{\partial\,z}\int\limits_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\,\Phi_{1I}(x,y,z,\tau)}{\partial\,z}d\,\tau+\Omega\frac{\partial}{\partial\,x}\int\limits_0^t\nabla_S\mu\,(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]d\,W\,d\,\tau$$

$$+\Omega\frac{\partial}{\partial\,y}\int\limits_0^t\nabla_S\mu\,(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]d\,W\;d\,\tau+\int\limits_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\,\tau$$

$$+\int\limits_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\,\tau+f_{\Phi_I}(x,y,z) \qquad (5e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial\,x}\int\limits_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\,\Phi_{1V}(x,y,z,\tau)}{\partial\,x}d\,\tau+\frac{\partial}{\partial\,y}\int\limits_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\,\Phi_{1V}(x,y,z,\tau)}{\partial\,y}d\,\tau$$

$$+\frac{\partial}{\partial\,z}\int\limits_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\,\Phi_{1V}(x,y,z,\tau)}{\partial\,z}d\,\tau+\Omega\frac{\partial}{\partial\,x}\int\limits_0^t\nabla_S\mu\,(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]d\,W\,d\,\tau$$

$$+\Omega\frac{\partial}{\partial\,y}\int\limits_0^t\nabla_S\mu\,(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]d\,W\;d\,\tau+\int\limits_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\,\tau$$

$$+\int\limits_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\,\tau+f_{\Phi_V}(x,y,z).$$

Average values of the second-order approximations of required approximations by using the following standard relation [26]

$$\alpha_{2\rho}=\frac{1}{\Theta L_xL_yL_z}\int\limits_0^{\Theta}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]d\,z\,d\,y\,d\,x\,d\,t\,. \qquad (12)$$

Substitution of the relations (1e), (3e), (5e) into relation (12) gives us possibility to obtain relations for required average values $\alpha_{2\rho}$

$$\alpha_{2C}=0,\ \alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0,\ \alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta\,a_3b_2+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2\,S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}}, \qquad (13)$$

where $b_4=\frac{S_{IV00}^2S_{VV00}}{\Theta L_xL_yL_z}-\frac{S_{VV00}^2S_{II00}}{\Theta L_xL_yL_z}$,

$$b_3=-\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+\right.$$

$$2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\Big)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3},$$

$$b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\Big(C_V+$$

$$S_{VV02}+S_{IV11}\Big)-\left(S_{IV10}-2S_{VV01}+\Theta L_xL_yL_z\right)^2+\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)+\frac{S_{IV00}}{\Theta L_xL_yL_z}\Big(S_{IV01}+$$

$$2S_{II10}+2S_{IV01}+\Theta L_xL_yL_z\Big)\left(2S_{VV01}+\Theta L_xL_yL_z+S_{IV10}\right)-\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(C_V-S_{VV02}-S_{IV11}\right)+\frac{C_IS_{IV00}^2}{\Theta^2L_x^2L_y^2L_z^2}-$$

$$2S_{IV01}\frac{S_{IV10}S_{IV00}}{\Theta L_xL_yL_z},\ b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)+\frac{S_{IV01}}{\Theta L_xL_yL_z}\Big(\Theta L_xL_yL_z+$$

$$2S_{II10}+S_{IV01}\Big)\left(2S_{VV01}+S_{IV10}+\Theta LL_yL_z\right)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}-\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(3S_{IV01}+2S_{II10}+\Theta L_xL_yL_z\right)\Big(C_V-$$

$$S_{VV02}-S_{IV11}\Big)+2C_IS_{IV00}S_{IV01},$$

$$b_0=\frac{S_{II00}}{\Theta L_xL_yL_z}\left(S_{IV00}+S_{VV02}\right)^2-S_{IV01}\frac{\Theta L_xL_yL_z+2S_{II10}+S_{IV01}}{L_xL_yL_z\Theta}\Big(C_V-$$

$$S_{VV02}-S_{IV11}\Big)+2C_IS_{IV01}^2-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right),$$

$$C_I=\frac{\alpha_{1I}\alpha_{1V}S_{IV00}}{\Theta L_xL_yL_z}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-\frac{S_{IV11}}{\Theta L_xL_yL_z},$$

$$E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\ C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}$$

$$-S_{IV11},\ F=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},\ r=\frac{\Theta^3b_2}{24b_4^2}\left(4b_0-\Theta L_xL_yL_z\frac{b_1b_3}{b_4}\right)-\frac{\Theta^3b_2^3}{54b_4^3}-$$

$$b_0\frac{\Theta^2}{8b_4^2}\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-L_x^2L_y^2L_z^2\frac{\Theta^4b_1^2}{8b_4^2},\ s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}.$$

Next, we determine solutions of Eqs.(10), i.e. components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections we replace the required functions in the right sides of the equations by their not yet known average values α_i. The substitution leads to the following result

$$\rho(z)\frac{\partial^2u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},$$

$$\rho(z)\frac{\partial^2u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},$$

$$\rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}. \tag{14}$$

Integration of the left and the right sides of the above relations on time *t* leads to the following result

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta, \tag{15}$$

$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined by using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [26]. The replacement leads to the following result

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}$$

$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+$$

$$+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y} \tag{16}$$

$$\left.-K(z)\right\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)d\tau\,d\vartheta-\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\right.$$

$$+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}$$

$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{6[1+\sigma(z)]}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\right\}.$$

Integration of the left and right sides of the above relations on time *t* leads to the following result

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\cdot$$

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$$\cdot\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int\limits_0^t\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\cdot$$

$$\cdot\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int\limits_0^t\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}-\frac{\partial}{\partial x}\int\limits_0^t\int\limits_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\cdot$$

$$\cdot\,K(z)\frac{\beta(z)}{\rho(z)}-\frac{1}{\rho(z)}\frac{\partial^2}{\partial x^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\cdot$$

$$\cdot\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta\cdot$$

$$\cdot\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0x}$$

$$-\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]+K(z)\frac{\beta(z)}{\rho(z)}\cdot$$

$$\cdot\frac{\partial}{\partial x}\int\limits_0^\infty\int\limits_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta \quad (16)$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\right]$$

$$+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2}{\partial y^2}\int\limits_0^t\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int\limits_0^t\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}$$

$$\cdot\int\limits_0^t\int\limits_0^\vartheta T\,(x,y,z,\tau)d\tau\,d\vartheta-\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\right]$$

$$-\frac{1}{\rho(z)}\left\{\frac{E(z)}{6[1+\sigma(z)]}-K(z)\right\}\frac{\partial^2}{\partial y\partial z}\int\limits_0^t\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\int\limits_0^\infty\int\limits_0^\vartheta T\,(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$-\frac{K(z)}{\rho(z)}\times\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)d\tau\,d\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int\limits_0^\infty\int\limits_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{5E(z)}{12[1+\sigma(z)]}\right.$$

$$\left.+K(z)\right\}-\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int\limits_0^\infty\int\limits_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}$$

$$-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial y\partial z}\int\limits_0^\infty\int\limits_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right.$$

$$\left.+\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial}{\partial z}\left\{\left[\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]K(z)\right\}\frac{1}{\rho(z)}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\right.$$

$$\cdot\left[6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right.$$

$$\left.\left.-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}\,.$$

According to the theoretical research, the dopant concentration, radiation defect concentrations and displacement vector components are calculated using the second-order approximation method of the averaging of function corrections. This estimate is usually good enough to make a good analysis and get very useful results. All results were verified by comparison with numerical simulation results.

DISCUSSION

In this section, we analyse the kinetics of additive and radial defect redistribution during annealing and under the influence of mismatch-induced stress. A profile of additive concentrations in heterostructures is shown in Figures 2 and 3 are for diffusion and ion-type doping, respectively. These distributions are calculated for the effect of the additive diffusion coefficient in the epitaxial process greater than the substrate. The figure shows that the inhomogeneity of the heterostructure makes it possible to increase the compactness of the transistor. At the same time, it is seen that the contribution distribution of the doped part of the epitaxial layer is gradually increasing. The compact nature of transistors makes it possible to make them fast.

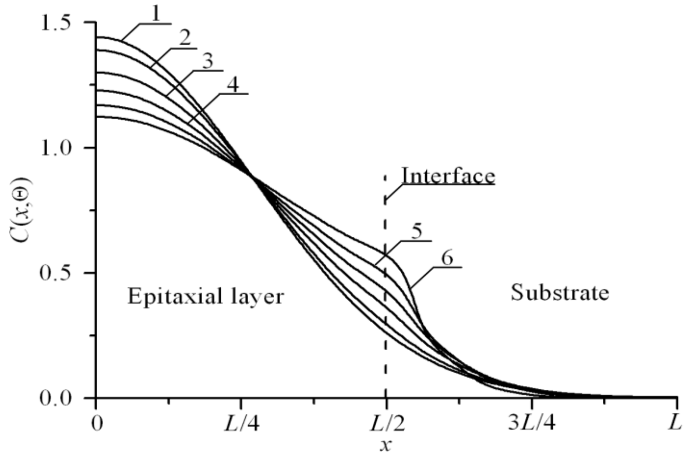

Fig. 2. Distributions of concentration of infused dopant in heterostructure from Fig. 1 in direction, which is perpendicular to interface between epitaxial layer substrate. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate

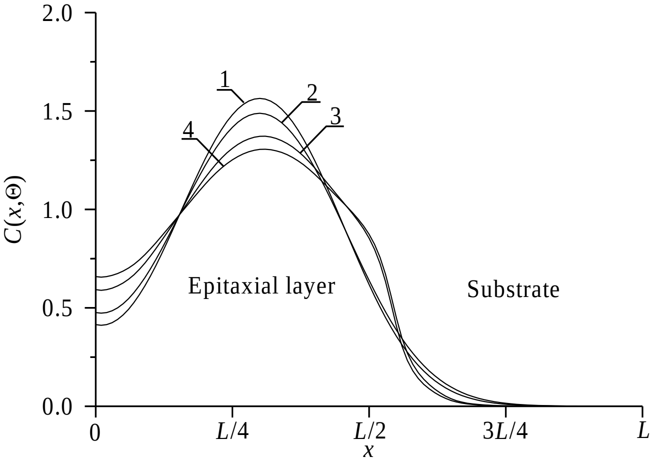

Fig. 3. Distributions of concentration of implanted dopant in heterostructure from Fig. 1 in direction, which is perpendicular to interface between epitaxial layer substrate. Curves 1 and 3 corresponds to annealing time $\Theta = 0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 corresponds to annealing time $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 correspond to homogenous sample. Curves 3 and 4 corresponds to heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate

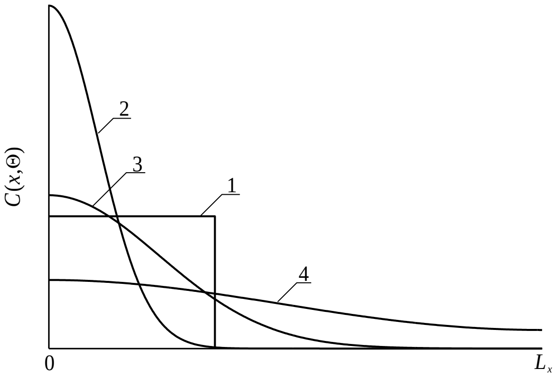

Fig. 4. Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

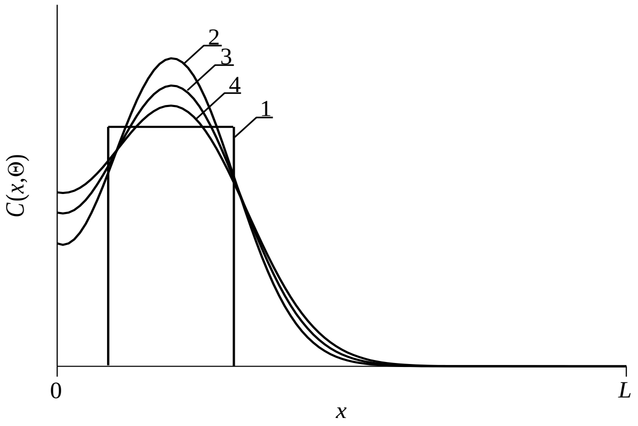

Fig. 5. Spatial distributions of dopant in heterostructure after ion implantation. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time.

The second consequence leads to a decrease in the local heating of materials during the operation of transistors or a reduction of their size to the maximum stable local overheating. However, in the framework of such a bipolar transistor fabrication method, the annealing of additives and/or electrical defects needs to be optimized. The reason for this optimization is as follows. If the annealing time is short, the additive does not provide any interaction between the heterostructure materials. In this case, a change in the additive concentration distribution cannot be detected. If the annealing time is long, the doping concentration distribution is also the same. We recently introduced a method to optimize the annealing time [15,25-32]. Within the framework of this process, we estimated the true distribution of additive concentrations with a stepwise function (see Figures 4 and 5). In addition, we determine the effective value of the annealing time by decreasing the mean square error below.

$$U=\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}\left[C\left(x,y,z,\Theta\right)-\psi\left(x,y,z\right)\right]d\,z\,d\,y\,d\,x, \tag{17}$$

where ψ (x,y,z) is the approximation function. The dependences of optimal values of annealing time on parameters are presented on Figs. 6 and 7 for diffusion and ion types of doping, respectively. It should be noted that it is necessary to anneal radiation defects after ion implantation. One could find the spreading of concentration of distribution of dopant during annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to additionally anneal the dopant. In that situation, the optimal value of additional annealing time of implanted dopant is smaller, than annealing time of infused dopant.

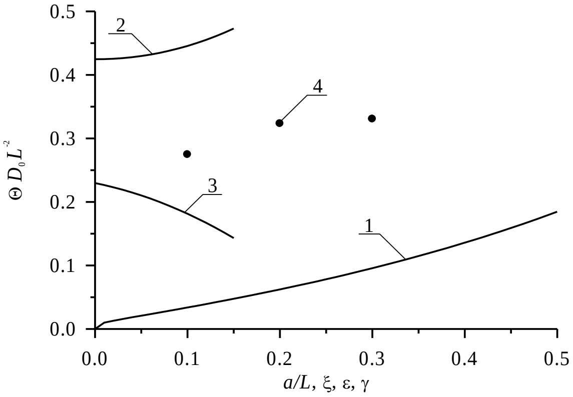

Fig.6. Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi=\gamma=0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi=\gamma=0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon=\xi=0$

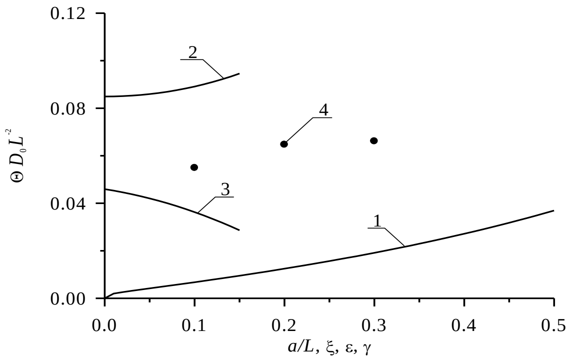

Fig.7. Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma =$ 0. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$

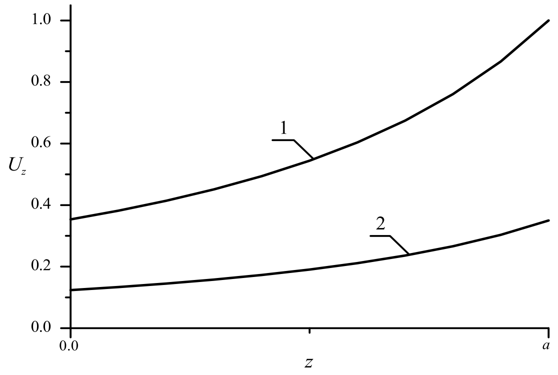

Fig. 8. Normalized dependences of component u_z of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

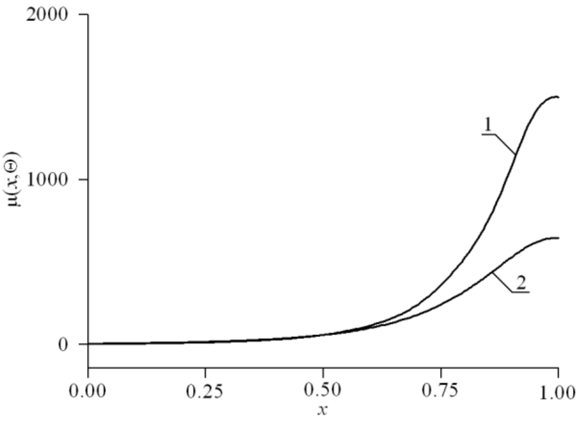

Fig. 9. Normalized distributions of charge carrier mobility in the considered heterostructure, the curve 1 corresponds to the heterostructure, which has been considered in Fig. 1. Curve 2 correspond to a homogenous material with averaged parameters of heterostructure from Fig. 1

Further we analyzed influence of relaxation of mechanical stress on distribution of dopant in doped areas of heterostructure. Under the following condition $\varepsilon_0 < 0$ one can find compression of distribution of concentration of dopant near interface between materials of heterostructure. On contrary (at $\varepsilon_0 > 0$) one can find spreading of distribution of concentration of dopant in this area. This changing of distribution of concentration of dopant could be at least partially compensated by using laser annealing [29]. This type of annealing gives us possibility to accelerate diffusion of dopant and another process in annealed area due to inhomogenous distribution of temperature and Arrhenius law. Moreover, accounting relaxation of mismatch-induced stress in heterostructure could leads to changing of optimal values of annealing time. Mismatch-induced stress could be used to increase density of elements of integrated circuits. On the other hand one could leads to generation dislocations of the discrepancy. Fig. 8 shows distributions of component of displacement vector, which is perpendicular to interface between layers of heterostructure.

CONCLUSION

In this paper, we simulate the implantation and redistribution of implanted dopants during the fabrication of the field-effect transistors (FET) in the framework of a absorptive single pole single throw switch, considering the stress caused by resting mismatch. We have developed recommendations for optimizing annealing to reduce the size of transistors and increase their speed. We provide recommendations to reduce the stress caused by inconsistencies. Analytical methods are presented to simulate diffusion and ion-type doping, including simultaneous spatial and temporal changes. At the same time, the method gives us the ability to consider the nonlinearity of the decision process.

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